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ABSTRACT. In this work we report our numerical modeling results of laser-generated transient inversion and capillary discharge X-ray lasers. In the search for more efficient X-ray lasers we look closely at other approaches in conjunction with experiments at LLNL. In the search for improved X-ray lasers we perform modeling and experimental investigations of low density targets including gas puff targets. We have found the importance of plasma kinetics in transient X-ray lasers by expanding the physical model beyond hydrodynamics approach with Particle In Cell (PIC) and Fokker-Planck codes. The evidence of the Langdon effect was inferred from the recent experimental data obtained with the Ni-like Pd X-ray laser. We continue modeling different kinds of capillary discharge plasma configurations directed toward shorter wavelength X-ray lasers, plasma diagnostics and other applications.

Keywords: x-ray lasers, laser plasma modeling, Z-pinch plasma modeling, interferometry

INTRODUCTION

Table-top X-ray lasers are becoming used more frequently in different applications as a result of their unique parameters and much improved affordability. Plasma diagnostics are an important area and it becomes clear that X-ray lasers are an object of research themselves. They represent a very sensitive, precise tool for extracting knowledge on different kinds of x-ray laser generating plasmas. This knowledge is highly valuable for advanced research in the areas of interaction of laser radiation with matter, plasma and other material diagnostics, validation of numerical codes, amongst others. Many groups have shown examples of extracting interesting new data utilizing the unique parameters of X-ray lasers. As an important example for our modeling, the very high, exponential sensitivity of the X-ray signal from gain and hence other parameters allowed us to register the Zeeman effect in capillary discharge X-ray laser [1]. Another example shows their advantages for diagnostics, where short wavelength X-ray lasers allowed the determination of the spatial distribution of ions (CI, CII and OI) and the spatial resolution of small perturbations in the plasma [2]. Again, the sensitivity of this approach has been defined by the exponential dependence of the absorption of the signal and reduced refraction due to shorter wavelengths. Below we show one more new example of utilization of exponential sensitivity, of the X-ray laser signal in this case, to the plasma conditions in laser plasmas. This has uncovered a point of interest in basic laser plasma physics.

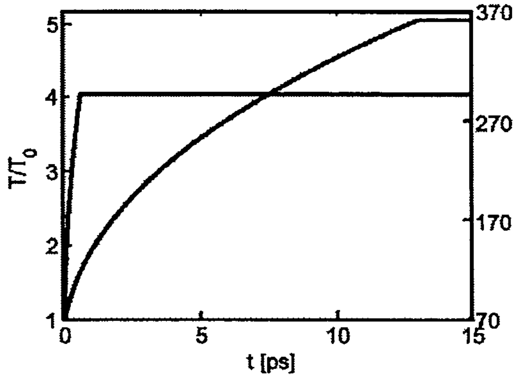
1. INFLUENCE OF PULSE DURATION ON TRANSIENT X-RAY LASERS

After the demonstration of transient X-ray lasers, experiments and modeling devoted substantial time for the optimization of the broad parameter space associated with the laser and target design. Among these, many different elements and their ion stages were investigated as well as different combinations of pre-pulse and short (main) pulse intensities, wavelengths and pulse durations. The theoretical and numerical research was devoted both to explain the data based on current knowledge in plasma physics, numerical analysis and computer capabilities and then to predict future directions of advanced research.

In recent work [3] one more interesting experimental investigation of Ni-like Pd X-ray laser output was performed as a function of short pulse duration. The latter was varied in a wide range from 0.6ps to 24 ps keeping laser energy constant ~5 J. Besides this, there exist several other experimental works, using the so-called prepulse technique, which utilize longer pulse durations of the order of ~100ps [4-6]. The numerical simulations of these experiments, both with 1ps and 100ps pulse durations using RADEX code led to a somewhat unusual conclusion. After successful reproduction and analysis of all of these experiments, we note that the prepulse technique lasers also operate in the transient regime. As an example, the transient gain of 'prepulse' lasers [5] is typically 3-5 times larger than quasi-steady state case. Transient inversion for the prepulse experiments does not give way to QSS as a result of the substantially, ~ 3 times, lower electron densities in the gain region compared to the picosecond experiments [3]. Additionally, the transient gain duration is expanded, or 'stretched', by ionization time from Zn- and Cu-like ionization stages to Ni- like stage to an additional ~50 ps. Note, that the larger density in the case of transient X-ray lasers with transverse pumping such as [3] is one of the reasons which decreases short pumping laser energy requirements and improves efficiency.

One area where there was found to be a contradiction was where numerical results for 0.6-1.2ps cases of [3] produced almost the same X-ray outputs as for 6ps case while the experiment demonstrated substantial decay there. The longer pulse cases with 13ps and 24 ps pulse durations also show decline in the x-ray laser output [3]. While clearly there is an effect due to the specific implementation of traveling wave with 7ps segments and increased mismatch of duration of transient gain which is of the order of 5-7 ps, the intensity decrease at shorter duration had no reasonable explanation. It is very interesting to note that 1-D hydrodynamics produce almost identical hydro parameters, within 1-2%, at the optimal plasma conditions for amplification for electron densities ~ 10^{20} cm^{-3} for all cases of pump laser pulse duration between 100 ps down to 100 fs. During analysis of plasma at these so different pulse durations we found that the differences in heat conduction and the influence of hot electrons on excitation and ionization during gain life-time ~7ps were estimated to be negligible. The heat conduction remains almost classical due to small ratio of electron mean free path to temperature gradient length at the distances as far as 30-70 microns from the target. Furthermore, it does not change the plasma parameters during very short gain duration of transient gain. The electron distribution function must relax to a Maxwellian quickly because of the relatively small pump fluxes ($<10^{15}$ W/cm^2) and much shorter electron collision time compared to the x-ray laser inversion life-time. Besides, all transient lasers work at relatively high temperatures when almost all electrons can take part in the excitation as opposed to the QSS schemes where only the electrons in the tails of distribution function are important, therefore requiring more electron-electron collisions to produce energetic particles. Hence, if we artificially introduce an amount of superthermal electrons (up to 20% at 1-3 keV, though we expect this amount to be much smaller at the reported experimental laser fluxes) there is no noticeable change in the key transient x-ray laser parameters. These include the gain, its duration and ionization state during amplification time and, most importantly, we find the gain-length product remains almost the same. The short duration of the transient inversion ensures ionization is almost frozen during amplification.

We found initially that the electron-ion $\tau_{ei} \sim 0.3$ ps and electron-electron $\tau_e = Z \times \tau_{ei} \sim 5$ ps collision times are not short compared to heating laser pulse duration when it shortens towards 0.6ps. A further piece of evidence here was reported by Langdon [7] who noticed that in high-Z plasmas due to the differences of noted electron collision times, the non-Maxwellian distribution function can be formed at laser fluxes Z times smaller than those corresponding to $v_q^2/v_e^2 \sim 1$. Here v_q is the maximum of the electron quiver velocity in laser field and v_e is electron thermal speed and this ratio defines heating per e-e collision compared to thermal energy or rate of deviation from Maxwellian distribution. As a result, the inverse-bremsstrahlung absorption will decrease by a factor ~ 0.5. This is exactly what is happening in our case where we have pulse duration shorter than ~5 ps and hence laser fluxes larger than $q \sim 1.2 \times 10^{14}$ W/cm^2 when $v_q \sim 3 \times 10^8$ cm/s and $v_e \sim 1.2 \times 10^9$ cm/s. Unfortunately treating these plasma kinetics processes numerically for 20 - 40 ps with modern direct PIC codes, even in 1-D, usually requires extremely long processing times not available at this moment [8]. Hence, we oriented on other approaches



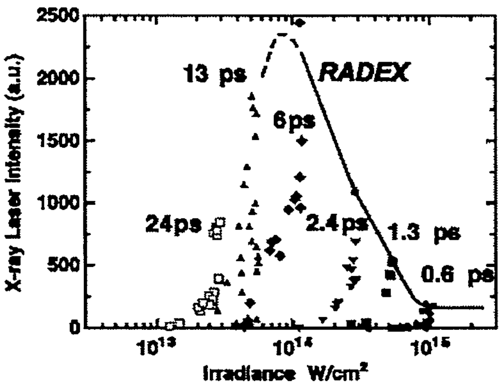


Fig. 1. PIC code calculations of electron temperature time evolution of plasma irradiated by two different pulses 0.6ps at 10^{15} W/cm^2 and 13ps at 5×10^{13} W/cm^2. Initial temperature was taken T_0=70eV which is not critical for final temperature.

Fig. 2. Experimental (squares and triangles) and numerical (solid line) X-ray laser signal vs pumping pulse duration. Dashed part of RADEX calculation curve takes into account the gain duration to traveling wave stepping mismatch

based on the Langevin equation solved within PIC algorithm and Fokker-Planck methods [9, 10]. The results of this calculation of electron temperature as a function of time for short laser pulse (0.6ps) and long laser pulse (13ps) duration cases based on the first approach are shown on Fig. 1. In these model runs, after laser deposition ends, the temperature remains at constant values because ionization, radiation losses and heat conduction were not included. RADEX calculations which include these processes, show that this approximation is valid to within better than 1% due to the short lasing time. The PIC simulation shows that electron distribution function becomes almost Maxwellian in just 2-3 ps after the laser pulse ends. As we see from Fig. 1, the electron temperatures are between 290eV and 360eV respectively for these limiting cases. Note, that in accordance with Langdon work, it stays almost the same with shorter than 0.6ps (270eV at 0.1 ps) pulse durations. For longer than 13 ps pulses it also does not change (with 24 ps temperature is 380 eV) because at lower fluxes the electron distribution function is Maxwellian. Based on these values, the atomic kinetics and ray-tracing code RADEX have well-reproduced the behavior of X-ray laser intensity versus flux of our previously reported work [3], see Fig. 2. Note that decreased at short pulse durations absorption not necessarily is bad for X-ray lasers, it might be even beneficiary, for example, in the cases of longitudinal pumping.

2. GAS PUFF TARGET X-RAY LASERS

Low density targets have attracted the attention of theory and experiment because of their appeal as an almost 'ideal' active medium for X-ray lasers [11,12]. At the same time the numerical calculations performed by our codes were in substantially disagreement with the experimental facts gathered during last decade. Besides both older experiments and our first attempts to create transient X-ray lasers with gas puff were lacking stability and explanations of failures. Recent experimental demonstration of picosecond-driven lasing in Ne-like Argon with gas puff targets [13] allowed us to obtain additional knowledge which pushed further research in this direction. One of the important facts which was determined in this work suggests that gas puff produces substantially larger densities than it was expected initially. For example quite large deflection angle of output radiation for this transient X-ray laser, of the order of 26-30 mrad, indicates that the optimal lasing conditions were formed at the electron density $\sim 4 \times 10^{20}$ cm^{-3} corresponding to particle pressures ~1.5atm, i.e. 3-5 times larger then it was expected on the nozzle exit. With such large densities it becomes clear why all other attempts to have lasing in Kr gas puff were unsuccessful since gain duration in Ni-like KrIX is too short, of the order of 1 ps, for used 7 ps stepping traveling wave scheme. Unlike the krypton, the gain duration in Ne-like Argon is several times larger, i.e. ~5-6 ps, with gain reaching 15-18 cm^{-1}, the value which fits well to experiment as it can be seen from initial slope of experimental curve up to 6 mm after which substantial refraction and potentially large inhomogeneities tamper the intensity [13]. Note also, that RADEX modeling suggests that the smaller measured transient gain and matched with traveling wave gain duration is an indication of transient gain formation in the conditions of simultaneous ionization (which is slow compared to inversion life-time) of lower Z ions before Ne-like ArIX. This somewhat similar to what takes place in prepulse technique transient lasers [4-6], and hence our low intensity prepulse most probably was not sufficient to ionize plasma up to Ne-like stage. The fact that best X-ray laser signal was obtained with the maximum energy of the available prepulse is also possible confirmation of this situation.

Since low density targets look promising as candidates toward shorter wavelength X-ray lasers, we performed calculations of the flow of the gas from the nozzle to get a better understanding of the plume expansion. Calculations were carried out for the momentum and energy transport in a gas flowing through a simple nozzle with straight parallel sides and into a vacuum chamber held at 1.32×10^{-6} atm (0.001 torr). The gas was assumed to behave as a continuum where the Navier-Stokes equations apply. The geometry and transport processes were assumed to be two-dimensional and steady. The initial gas source conditions were specified to be argon at 10 atm and 295 K. The two-equation k-ε turbulence model was used [14]. The Sutherland law was used for the temperature dependence of the viscosity, $\mu/\mu_0 = (T/T_0)^{1.5}(T_0+S)/(T+S)$, where for argon $\mu_0=2.125 \times 10^{-4}$ g/(cm·s), $T_0=273$ K and S = 144.4 K [15]. The thermal conductivity was obtained from the Prandtl number, $Pr = \mu c_p/k$, where for argon Pr = 0.685, and the specific heat was assumed to be constant at 5.27×10^6 erg/(g·K) [16]. The calculations were carried out using the commercial code, INCA [17], which is an upwind implicit Navier-Stokes finite volume code.

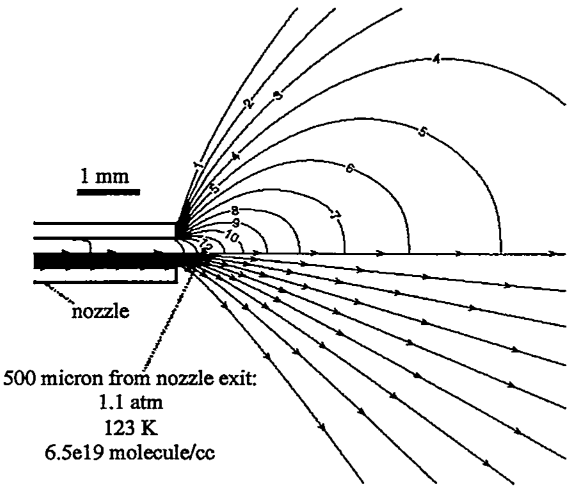


Fig. 3. The calculated gas flow (streamlines shown in the lower half of the figure) and density (density contours shown in the upper half of the figure) fields. Density for contour J is: $n_J = 1.45^{J-1} \cdot 10^{18}$ cm^{-3}.

Fig. 3 shows results for the gas flow and density fields. Streamlines are shown in the lower half of the figure and gas number density contours are shown in the upper half. The streamlines are nearly straight far from the nozzle exit, characteristic of a free jet in a low pressure environment. The density of the gas decreases with distance from the nozzle exit. The conditions at the laser target point, located on the nozzle centerline 500 μm from the nozzle exit, are 1.1 atm, 123 K and 6.5×10^{19} molecule/cc. As it can be seen from Fig.3, densities at the ~150 μm distance from the nozzle exit are approximately twice larger than deflection angle suggests, but since this is steady flow, these values give upper limit of expectable densities. These steady flow calculations qualitatively confirm the possibilities of larger gas densities at which our X-ray laser operates. Calculations of unsteady flow in which the density is varying depending on time delay from valve opening are under the progress.

3. LASER PLASMA DENSITY PROFILE MODELING

Both capillary discharge and transient table-top lasers are probing their ability to be used in applications to diagnose high density plasmas using the coherence properties. Several works were done recently to demonstrate interferometry utilizing these lasers [18,19]. Very valuable and surprising results were obtained already in the first experimental shots in the mentioned works and we used available numerical codes LASNEX and RADEX to evaluate obtained plasma parameters. The density profiles obtained in [18] show considerable inhomogeneities which look as suppression of density in the middle of laser focal spot. Though axial jets and filaments in plasma were reported in numerous experimental works earlier, the formation of inhomogeneities of density profile of such large magnitude at such relatively small laser fluxes has never seen before. The density, which is reaching critical one 10^{21} cm^{-3} for plasma heated by small-energy Nd-glass lasers at the 150-200 micron away from the 30 microns wide focal spot, may look at obvious contradiction with classical description of steady-state spherical plasma expansion under the influence of laser radiation [20] and world modeling practice. It is well established that at low laser fluxes plasma size is self-consistently adjusting to be of the order of radius R_0 of expansion (which is of the focal spot size in the case of plane target) or as small as just $\sim 1.2R_0$ in the case of higher fluxes when absorption takes place on critical surface. From that point it was clear that 1-D numerical model could not reproduce these dense peaks far from the laser spot. Our 1-D and 2-D modeling ruled out the possibility of ponderomotive force, saturation of electron heat conductivity, and additional pre-heat inside expanded plasma column far away from the surface as primary reasons for this density distribution with suppression.

After extensive analysis it was found that there is no contradiction with previous and current modeling. The minimum of the density, positioned on-axis along the normal to the target surface, corresponds to the regular expansion of illuminated part of plasma, the parameters of which are strongly governed via inverse bremsstrahlung absorption by the laser wavelength. This part of density profile fits well to both 1-D or 2-D hydrodynamics models description of sub-critical density and velocity evolution of plasmas under laser irradiation, while the unusually large density peaks accompanying the minimum are, in fact, formed outside the focal spot. These peaks represent colder material expansion and are formed later in time. They are produced by heating of material by XUV radiation emitted by the plasma and by lateral heat conduction from hot focal spot area. As it can be seen from Fig.4, the radiation and thermal conduction processes effectively increase ablation area by several times compared to initial one. Such off-spot energy

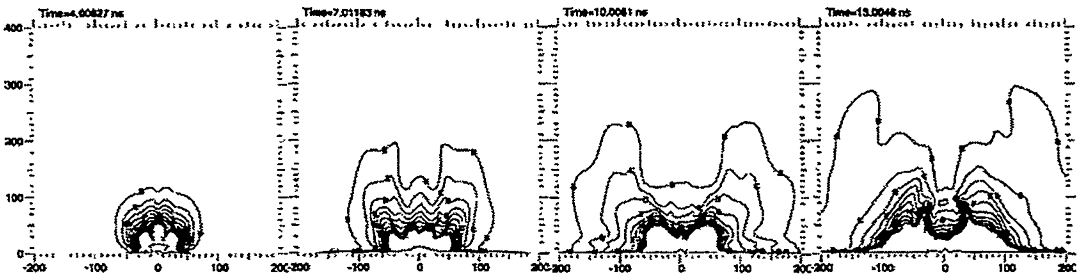


Fig. 4. Expansion of plasma stripe irradiated by 1 ω, 10ns FWHM, 10^{11} W/cm^2 laser pulse using line focus with focal spot line width 30μm. Density contours (A: 1×10^{19} cm^{-3}, B: 2×10^{19} cm^{-3} etc) shown at the moments as indicated on the top.

deposition represents a purely 2-D effect, and it does not depend directly on the laser wavelength. Because critical density for XUV radiation is much larger than it is for laser radiation $n_c = 10^{21}$ cm^{-3} the side peaks values are not limited by n_c and can grow even further.

Velocity of central and hotter on-axis part plasma, formed by large pressure gradient near the critical surface, achieves $(5\text{-}6) \times 10^6$ cm/s while in off-spot regions of smaller gradients it moves at several times lower speeds gradually increasing up to $(2\text{-}3) \times 10^6$ cm/s. Due to these speed differences the high-density plasma peaks do not appear or are less pronounced at earlier times up to ~6 ns of 10 ns laser pulse as shows LASNEX 2-D modeling (see Fig.4) and only at later times they are reaching distances ~100-200 microns from the target as observed in the experiment. The profiles with density suppression on axis inside of which plasma behaves like a jet were observed both with laser shots on a new target, as well as when the laser was shot two or more times in the same target position creating a crater 70-300 μm deep [18]. In the latter case, due to cumulation effect, the crater left by the previous shots may be an additional factor that contributes to increase the maximum density of the side lobes far from the target, what we are currently studying with LASNEX. Also under way are more detailed plasma simulations and comparisons with the experiments of spot geometry where laser fluxes were higher.

Qualitatively similar density bumps, but of much smaller magnitude were also revealed in another interferometric experiments with transient X-ray laser [19]. Due to lower Z, much shorter laser pulse duration and expansion time before probing, and wider spot size, they were less pronounced. Radex 1-D calculations of density agree well with this experimental data at the developed stages of laser expansion recorded along the normal in the middle of the target, since expansion in the normal to the surface can be well described in cylindrical or spherical 1-D approximation. But we also found different kind of discrepancy. We see substantially large disagreement in the expansion of plasma at the very initial times of laser deposition where probing shows that plasma succeeded to expand up to 150 microns during just first ~400ps from the origin of laser pulse. The plasma speeds in experiment in excess of $\sim 4 \times 10^7$ cm/s hence are several times larger than in modeling. These very uncommon differences, given pretty small laser fluxes 1.7×10^{11} W/cm^2 suggest that there may exist some additional physical mechanisms not covered by the numerical model which allowed to overheat small amount of vapors to unusually large temperatures >300eV at the rise of laser pulse. There also may exist a very long low-flux pre-pulse, but more probable explanation could be related to the reflection of X-rays due to substantial refraction in the region very close to the target which bended probing X-rays far away given a very steep density profile near the surface at the initial moments. Blocking these rays before they enter the plasma would probably easily remove or confirm this as a real reason. Since such kind of research is important for the development and validation of hydrocodes and plasma theory as well as for experiment and plasma diagnostics, we will be continuing comparisons of simulations with this encouraging results obtained with X-ray laser interferometric methods.

4. HIGH CURRENT CAPILLARY DISCHARGE X-RAY LASERS

After demonstration of small and efficient capillary X-ray lasers there exists a great interest in extending this approach to shorter wavelengths. Substantial experimental and theoretical efforts were devoted in this direction in recent years. A series of computer simulations and experiments of spectroscopic diagnostics of plasma parameters using Argon gas were completed recently to investigate the plasma dynamics, the range of temperatures, densities, to check plasma stability at higher currents and validate numerical model assumptions [21]. The N_e ~2-3 10^{20} cm^{-3} electron density and temperatures T_e ~250eV (up to 500 eV at the initial part of collapse) were inferred based on agreement with pinhole imaging experiments (plasma size, its temporal evolution) and spectroscopic measurements (spectra in 100-200A region and relative abundances of higher–Z species). We report here the results of extension of well developed Ne-like X-ray laser scheme to higher Z Ni-like ions with the wavelengths range around ~100 Å [22]. The temperatures and densities of the order of 200eV and (1-3) $\times 10^{20}$ cm^{-3} and higher achievable in 200 kA discharge the atomic elements suitable for lasing range from A=42 to 50. Given the small or even negative (i.e. absorption) QSS gain in light elements in Ni-like ions, the noticeable contribution of transient gain is predicted in the capillary discharge plasma during fast ionization of ions. Of course because transient gain duration is substantially shorter than ionization time of Zn- and Cu-like ions in this case of not too overheated plasma (of the order of 0.1-1 ns) the maximum values are not expected to be very high. For Ni-like CdXXI, for example, the gain is calculated ~1-2 cm^{-1}, and potentially >3 cm^{-1} with further optimization of parameters and at higher electrical currents, which seems to be in qualitative agreement with current experimental data. Larger gain coefficients were computed is possible to achieve in Ni-like AgXX and PdXIX which requires further investigation.

5. ACKNOWLEDGMENTS

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